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Final-state rescattering and SU(3) symmetry breaking in $B \to DK$ and $B \to DK^*$ decays

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Abstract. The first observation of the $\bar{B}^0_d \to D^0\bar{K}^0$ and $\bar{B}^0_d \to D^0\bar{K}^{*0}$ transitions by the Belle Collaboration allows us to do a complete isospin analysis of the $B \to DK^{(*)}$ decay modes. We find that their respective isospin phase shifts are very likely to lie in the ranges $37^\circ \leq (\phi_1-\phi_0)_{DK} \leq 63^\circ$ (or around 50°) and $25^\circ \leq (\phi_1-\phi_0)_{DK^*} \leq 50^\circ$ (or around 35°), although the possibility $(\phi_1-\phi_0)_{DK} = (\phi_1-\phi_0)_{DK^*} = 0^\circ$ cannot be ruled out at present. Thus significant final-state rescattering effects possibly exist in such exclusive $|\Delta B| = |\Delta C| = |\Delta S| = 1$ processes. We determine the spectator and color-suppressed spectator quark-diagram amplitudes of the $B \to DK$ and $B \to DK^*$ decays, and compare them with the corresponding quark-diagram amplitudes of the $B \to D\pi$ and $B \to D\rho$ decays. The effects of SU(3) flavor symmetry breaking are in most cases understandable in the factorization approximation, which works for the individual isospin amplitudes. Very instructive predictions are also obtained for the branching fractions of rare $\bar{B}^0_d \to \bar{D}^0\bar{K}^{(*)0}$, $B^-_u \to \bar{D}^0K^{(*)-}$ and $B^-_u \to D^-\bar{K}^{(*)0}$ transitions.

1 Introduction

The major goal of B-meson factories is to test the Kobayashi–Maskawa mechanism of CP violation within the standard model and to detect possible new sources of CP violation beyond the standard model. So far the CP-violating asymmetry in B^0_d versus $\bar{B}^0_d \to J/\psi K_S$ decays has been unambiguously measured at KEK and SLAC [1], and the experimental result is very well compatible with the standard-model expectation. Further experiments will provide much more data on CP violation in other decay modes of the B mesons, from which one may cross-check the consistency of the Kobayashi–Maskawa picture and probe possible new physics.

Two-body non-leptonic decays of the type

$$\bar{B}^0_d \to X^+Y^- , \quad \bar{B}^0_d \to X^0\bar{Y}^0 , \quad B^-_u \to X^0Y^-$$

and their CP-conjugate modes, which occur only via the tree-level quark diagrams and can be related to one another via the isospin triangles, have been of great interest in B physics for a stringent test of the factorization hypothesis, a quantitative analysis of final-state interactions [2], and a clean determination of the CP-violating phases [3]. The typical examples are $X = (D, D^*)$ and $Y = (\pi, \rho)$ as well as $X = (D, D^*)$ and $Y = (K, K^*)$, associated respectively with the weak phases $(2\beta+\gamma)$ [4] and γ [5]. Once the decay rates of $\bar{B}^0_d \to X^+Y^-$, $\bar{B}^0_d \to X^0\bar{Y}^0$ and $B^-_u \to X^0Y^-$ are all measured to an acceptable degree of accuracy, one may construct the isospin relations of their transition amplitudes and determine the relevant strong and weak phases. Among the three channels under discussion, the neutral one, $\bar{B}^0_d \to X^0\bar{Y}^0$, is color-suppressed and has the smallest branching ratio. Hence it is most difficult to measure $\bar{B}^0_d \to X^0\bar{Y}^0$ in practice, even at B-meson factories. Indeed $\bar{B}^0_d \to D^0\pi^0$ and $D^{*0}\pi^0$ decays were not observed until 2001 [6]. The isospin analysis of three $B \to D^{(*)}\pi$ modes [7] indicates that they involve significant final-state rescattering effects.

Recently the Belle Collaboration [8] has reported the first observation of $\bar{B}^0_d \to D^0\bar{K}^0$ and $\bar{B}^0_d \to D^0\bar{K}^{*0}$ decays. Their branching fractions are found to be

$$\mathcal{B}^{DK}_{00} = \left(5.0^{+1.3}_{-1.2} \pm 0.6\right) \times 10^{-5} ,$$
$$\mathcal{B}^{DK^*}_{00} = \left(4.8^{+1.1}_{-1.0} \pm 0.5\right) \times 10^{-5} , \qquad (1)$$

respectively. In comparison, the branching fractions of $\bar{B}^0_d \to D^+K^{(*)-}$ and $B^-_u \to D^0K^{(*)-}$ decays are [9]

$$\mathcal{B}^{DK}_{+-} = (2.0 \pm 0.6) \times 10^{-4} ,$$
$$\mathcal{B}^{DK^*}_{+-} = (3.7 \pm 1.8) \times 10^{-4} , \qquad (2)$$

and

$$\mathcal{B}^{DK}_{0-} = (3.7 \pm 0.6) \times 10^{-4} ,$$
$$\mathcal{B}^{DK^*}_{0-} = (6.1 \pm 2.3) \times 10^{-4} . \qquad (3)$$

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We see that $\mathcal{B}^{DK}_{+-} \sim \mathcal{B}^{DK}_{0-} > \mathcal{B}^{DK}_{00}$ and $\mathcal{B}^{DK^*}_{+-} \sim \mathcal{B}^{DK^*}_{0-} > \mathcal{B}^{DK^*}_{00}$ do hold, as naively expected. With the help of the new experimental data, one is now able to analyze the isospin relations for the amplitudes of $B \to DK^{(*)}$ decays in a more complete way than before (see, e.g., [10,11]). Then it becomes possible to examine whether final-state interactions are significant or not in such exclusive $|\Delta B| = |\Delta C| = |\Delta S| = 1$ transitions.

The purpose of this paper is four-fold. First, we make use of current experimental data to determine the isospin amplitudes and their relative phase for $B \to DK^{(*)}$ transitions. We find that the isospin phase shifts in $B \to DK$ and $B \to DK^*$ decays are very likely to lie in the ranges $37^\circ \leq (\phi_1 - \phi_0)_{DK} \leq 63^\circ$ (or around 50°) and $25^\circ \leq (\phi_1 - \phi_0)_{DK^*} \leq 50^\circ$ (or around 35°), although the possibility $(\phi_1 - \phi_0)_{DK} = (\phi_1 - \phi_0)_{DK^*} = 0^\circ$ cannot be ruled out at the moment. Hence significant final-state rescattering effects possibly exist in these interesting $|\Delta B| = |\Delta C| = |\Delta S| = 1$ processes. Second, we carry out a quark-diagram analysis of $B \to DK^{(*)}$ transitions and determine the amplitudes of their spectator and color-suppressed spectator diagrams. Our result is model-independent, thus it can be used to test the predictions from specific models of hadronic matrix elements. Third, we compare the quark-diagram amplitudes of $B \to DK$ and $B \to DK^*$ with those of $B \to D\pi$ and $B \to D\rho$ to examine the SU(3) flavor symmetry. We find that the effects of SU(3) symmetry breaking are in most cases understandable in the factorization approximation, which works well for the individual isospin amplitudes. Finally, we present very instructive predictions for the branching fractions of the rare $\bar{B}^0_d \to \bar{D}^0 \bar{K}^{(*)0}$, $B^-_u \to \bar{D}^0 K^{(*)-}$ and $B^-_u \to D^- \bar{K}^{(*)0}$ decays.

2 Isospin analysis

Let us begin with an isospin analysis of the $\bar{B}^0_d \to D^+ K^-$, $\bar{B}^0_d \to D^0 \bar{K}^0$ and $B^-_u \to D^0 K^-$ decays. As D and K are isospin 1/2 mesons, a final state DK may be in either the $I = 1$ or the $I = 0$ configuration. To be specific, $D^0 K^-$ is a pure $I = 1$ state and involves a single rescattering phase. In contrast, $D^+ K^-$ and $D^0 \bar{K}^0$ are combinations of $I = 1$ and $I = 0$ states, which mix under rescattering. The amplitudes of three decay modes can therefore be expressed as

$$\begin{aligned} A^{DK}_{+-} &= \frac{1}{2} A^{DK}_1 + \frac{1}{2} A^{DK}_0 \; , \\ A^{DK}_{00} &= \frac{1}{2} A^{DK}_1 - \frac{1}{2} A^{DK}_0 \; , \\ A^{DK}_{0-} &= A^{DK}_1 \end{aligned} \qquad (4)$$

in terms of two isospin amplitudes A^{DK}_1 and A^{DK}_0. The weak phases of A^{DK}_1 and A^{DK}_0 are identical to $\arg(V_{cb} V^*_{us})$ [10], where V_{cb} and V_{us} are the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements. The strong phases of A^{DK}_1 and A^{DK}_0, denoted respectively as ϕ_1 and ϕ_0, are in general different from each other. The branching fractions of the $\bar{B}^0_d \to D^+ K^-$, $\bar{B}^0_d \to D^0 \bar{K}^0$ and $B^-_u \to D^0 K^-$ transitions read

$$\begin{aligned} \mathcal{B}^{DK}_{+-} &= \frac{p_{DK}}{8\pi M^2_B} \left| A^{DK}_{+-} \right|^2 \tau_0 \; , \\ \mathcal{B}^{DK}_{00} &= \frac{p_{DK}}{8\pi M^2_B} \left| A^{DK}_{00} \right|^2 \tau_0 \; , \\ \mathcal{B}^{DK}_{0-} &= \frac{p_{DK}}{8\pi M^2_B} \left| A^{DK}_{0-} \right|^2 \tau_\pm \; , \end{aligned} \qquad (5)$$

where τ_0 (or $\tau_\pm$) is the lifetime of B^0_d or $\bar{B}^0_d$ (or $B^\pm_u$), and

$$p_{DK} = \frac{1}{2M_B} \times \sqrt{\left[M^2_B - (M_D + M_K)^2\right] \left[M^2_B - (M_D - M_K)^2\right]} \qquad (6)$$

is the c.m. momentum of the D and K mesons. Note that the mass difference between neutral and charged B, D or K mesons is tiny and has been neglected in (5) and (6). From (4) and (5), we obtain

$$\begin{aligned} \left|A^{DK}_1\right| &= 2M_B \sqrt{\frac{2\pi\kappa \mathcal{B}^{DK}_{0-}}{\tau_0 p_{DK}}} \; , \\ \left|A^{DK}_0\right| &= 2M_B \sqrt{\frac{2\pi \left[2\left(\mathcal{B}^{DK}_{+-} + \mathcal{B}^{DK}_{00}\right) - \kappa \mathcal{B}^{DK}_{0-}\right]}{\tau_0 p_{DK}}} \; , \end{aligned} \qquad (7)$$

with $\kappa \equiv \tau_0/\tau_\pm \approx 0.92$ [9], and

$$\cos(\phi_1 - \phi_0)_{DK} = \frac{\mathcal{B}^{DK}_{+-} - \mathcal{B}^{DK}_{00}}{\sqrt{\kappa \mathcal{B}^{DK}_{0-} \left[2\left(\mathcal{B}^{DK}_{+-} + \mathcal{B}^{DK}_{00}\right) - \kappa \mathcal{B}^{DK}_{0-}\right]}} \; . \qquad (8)$$

If final-state interactions were insignificant, one would have $\cos(\phi_1 - \phi_0)_{DK} \approx 1$. In this case, the naive factorization approximation could be applied to the *overall* amplitudes of the three decay modes under discussion.

To illustrate this, we take the experimental values of $\mathcal{B}^{DK}_{+-}$, $\mathcal{B}^{DK}_{00}$ and $\mathcal{B}^{DK}_{0-}$ in (1), (2) and (3) to calculate the isospin amplitudes A^{DK}_1 and A^{DK}_0. The values of the other input parameters in (7) and (8) can be found from [9]. Our numerical results are shown in Fig. 1, from which we obtain 1.93×10^{-7} GeV $\leq |A^{DK}_1| \leq 2.28 \times 10^{-7}$ GeV, 0.44×10^{-7} GeV $\leq |A^{DK}_0| \leq 2.19 \times 10^{-7}$ GeV, and $0.41 \leq \cos(\phi_1 - \phi_0)_{DK} \leq 1$. Although the possibility $(\phi_1 - \phi_0)_{DK} = 0^\circ$ cannot be excluded for the time being [12], Fig. 1 indicates that this isospin phase shift is most likely to lie in the range $37^\circ \leq (\phi_1 - \phi_0)_{DK} \leq 63^\circ$. In particular, the central values of $|A^{DK}_1|$, $|A^{DK}_0|$ and $(\phi_1 - \phi_0)_{DK}$ read

$$\begin{aligned} \left|A^{DK}_1\right| &\approx 2.1 \times 10^{-7}\,\text{GeV} \; , \\ \left|A^{DK}_0\right| &\approx 1.4 \times 10^{-7}\,\text{GeV} \; , \end{aligned} \qquad (9)$$

and

$$(\phi_1 - \phi_0)_{DK} \approx 50^\circ \; . \qquad (10)$$

Our results imply that significant final-state rescattering effects are very likely to exist in $B \to DK$ decays. Hence

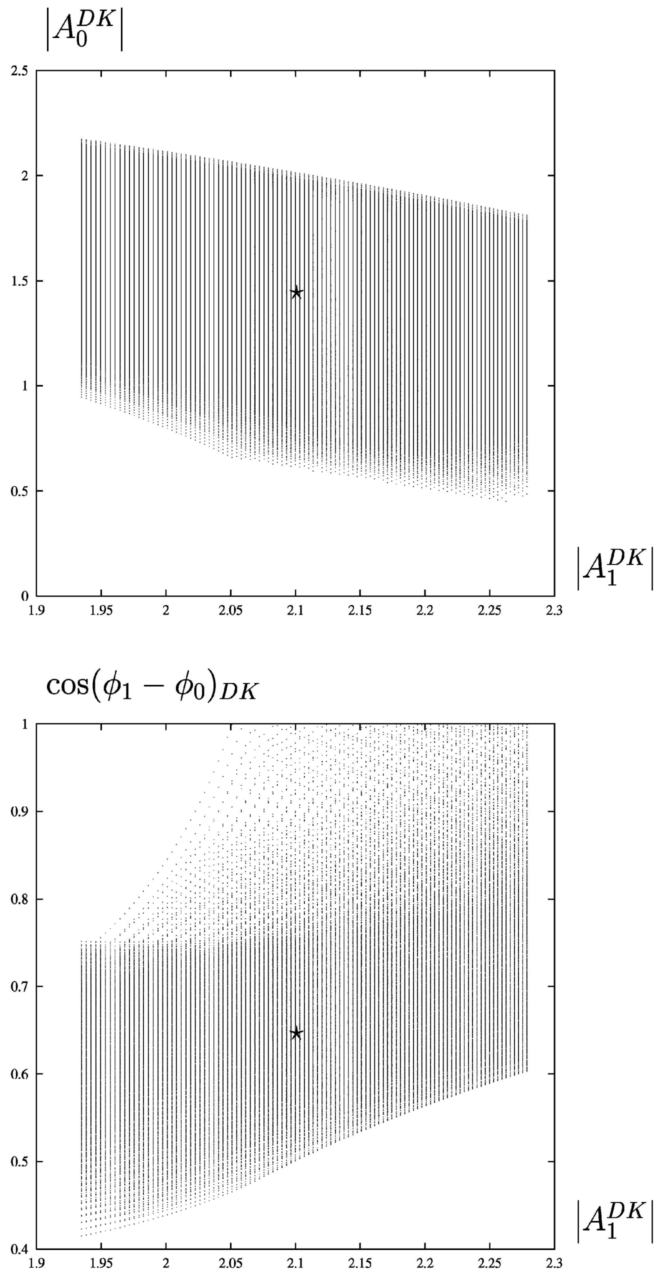

Fig. 1. The ranges of $|A_1^{DK}|$, $|A_0^{DK}|$ and $\cos(\phi_1 - \phi_0)_{DK}$ allowed by the current experimental data, where $\star$ indicates the result obtained from the central values of $\mathcal{B}_{+-}^{DK}$, $\mathcal{B}_{00}^{DK}$ and $\mathcal{B}_{0-}^{DK}$

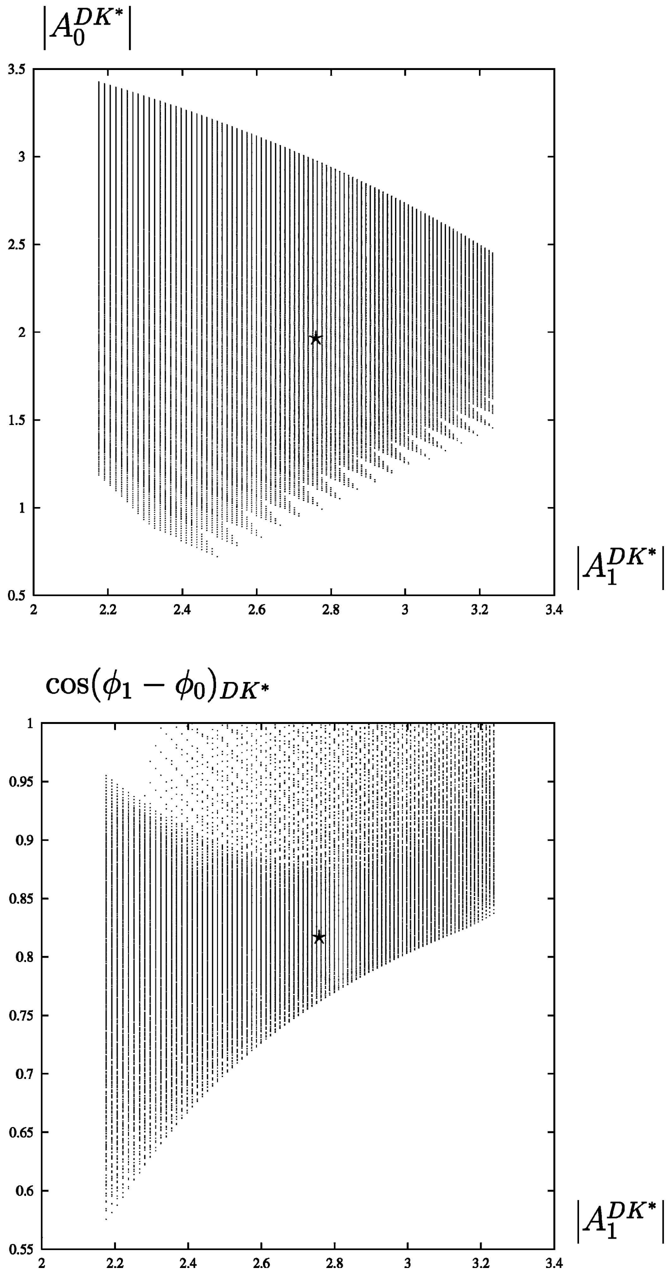

Fig. 2. The ranges of $|A_1^{DK^*}|$, $|A_0^{DK^*}|$ and $\cos(\phi_1 - \phi_0)_{DK^*}$ allowed by the current experimental data, where $\star$ indicates the result obtained from the central values of $\mathcal{B}_{+-}^{DK^*}$, $\mathcal{B}_{00}^{DK^*}$ and $\mathcal{B}_{0-}^{DK^*}$

the neutral mode $\bar{B}_d^0 \to D^0 \bar{K}^0$ may not be strongly suppressed, even though A_1^{DK} and A_0^{DK} are comparable in magnitude.

As the $B \to DK^*$ decays have the same isospin relations as $B \to DK$ decays, one may carry out an analogous analysis to determine the isospin amplitudes of the former ($A_1^{DK^*}$ and $A_0^{DK^*}$) by use of the experimental values of $\mathcal{B}_{+-}^{DK^*}$, $\mathcal{B}_{00}^{DK^*}$ and $\mathcal{B}_{0-}^{DK^*}$ in (1), (2) and (3). Our results are presented in Fig. 2, from which $2.17 \times 10^{-7}\,\mathrm{GeV} \leq |A_1^{DK^*}| \leq 3.23 \times 10^{-7}\,\mathrm{GeV}$, $0.71 \times 10^{-7}\,\mathrm{GeV} \leq |A_0^{DK^*}| \leq 3.42 \times 10^{-7}\,\mathrm{GeV}$, and $0.58 \leq \cos(\phi_1 - \phi_0)_{DK^*} \leq 1$ are directly obtainable. Note again that the possibility $(\phi_1 - \phi_0)_{DK^*} = 0°$ cannot be ruled out at the moment [12], but Fig. 2 indicates that this isospin phase shift is most likely to lie in the range $25° \leq (\phi_1 - \phi_0)_{DK^*} \leq 50°$. The central values of $|A_1^{DK^*}|$, $|A_0^{DK^*}|$ and $(\phi_1 - \phi_0)_{DK^*}$ are actually

$$\left|A_1^{DK^*}\right| \approx 2.8 \times 10^{-7}\,\mathrm{GeV}\,,$$
$$\left|A_0^{DK^*}\right| \approx 1.9 \times 10^{-7}\,\mathrm{GeV}\,, \tag{11}$$

and

$$(\phi_1 - \phi_0)_{DK^*} \approx 35°\,. \tag{12}$$

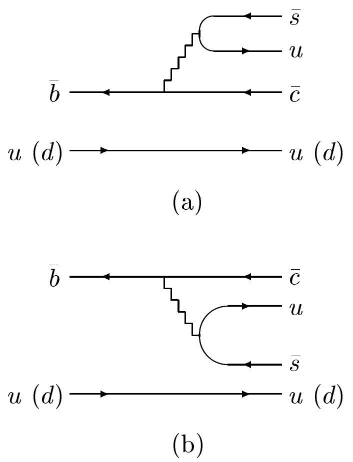

Fig. 3a,b. The quark diagrams responsible for the $B \to DK$ and $B \to DK^*$ decays: **a** the spectator; **b** the color-suppressed spectator

It becomes clear that large final-state interactions are also likely to exist in $B \to DK^*$ decays. Our results imply that the naive factorization approximation should not be *directly* applied to both $B \to DK$ and $B \to DK^*$ transitions. A proper treatment of such exclusive processes has to take account of final-state rescattering effects at the hadron level.

3 Quark-diagram analysis

Now we turn to the description of the $B \to DK$ and $B \to DK^*$ decays in the language of quark diagrams, which is sometimes more intuitive and instructive than the isospin language. As shown in Fig. 3, these processes occur only via two tree-level quark diagrams: one is the spectator diagram and the other is the color-suppressed spectator diagram. After the CKM matrix elements (V_{cb} and V^*_{us}) are factored out, the remaining parts of the quark-diagram amplitudes in Fig. 3a,b can be defined under isospin symmetry as T_{DK} (or T_{DK^*}) and T'_{DK} (or T'_{DK^*}), respectively. The relations between quark-diagram amplitudes and isospin amplitudes of $B \to DK$ transitions are

$$
\begin{aligned}
A^{DK}_1 &= V_{cb} V^*_{us} \left(T_{DK} + T'_{DK}\right) \mathrm{e}^{\mathrm{i}\phi_1} \; , \\
A^{DK}_0 &= V_{cb} V^*_{us} \left(T_{DK} - T'_{DK}\right) \mathrm{e}^{\mathrm{i}\phi_0} \; .
\end{aligned}
\tag{13}
$$

Without loss of generality, T_{DK} and T'_{DK} can be arranged to be real and positive. In addition, $T_{DK} > T'_{DK}$ is expected to hold, as the latter is color-suppressed. The explicit expressions of T_{DK} and T'_{DK} can straightforwardly be derived from (4), (5) and (13). The result is

$$
\begin{aligned}
T_{DK} &= \frac{M_B}{|V_{cb} V^*_{us}|} \sqrt{\frac{2\pi}{\tau_0 p_{DK}}} \\
&\quad \times \left[\sqrt{\kappa \mathcal{B}^{DK}_{0-}} + \sqrt{2\left(\mathcal{B}^{DK}_{+-} + \mathcal{B}^{DK}_{00}\right) - \kappa \mathcal{B}^{DK}_{0-}} \right] , \\
T'_{DK} &= \frac{M_B}{|V_{cb} V^*_{us}|} \sqrt{\frac{2\pi}{\tau_0 p_{DK}}} \\
&\quad \times \left[\sqrt{\kappa \mathcal{B}^{DK}_{0-}} - \sqrt{2\left(\mathcal{B}^{DK}_{+-} + \mathcal{B}^{DK}_{00}\right) - \kappa \mathcal{B}^{DK}_{0-}} \right] .
\end{aligned}
\tag{14}
$$

Of course, it is easy to obtain the similar expressions for T_{DK^*} and T'_{DK^*}, the quark-diagram amplitudes of the $B \to DK^*$ decay modes.

It is worth emphasizing that (14) allows us to determine T_{DK} (or T_{DK^*}) and T'_{DK} (or T'_{DK^*}) in a model-independent way. On the other hand, these quark-diagram amplitudes can be evaluated using the factorization approximation in specific models of hadronic matrix elements [2]. Thus the validity of the factorization hypothesis for $B \to DK$ and $B \to DK^*$ decays is experimentally testable[1].

For illustration, we calculate (T_{DK}, T'_{DK}) and (T_{DK^*}, T'_{DK^*}) numerically by use of the central values of $(\mathcal{B}^{DK}_{+-}, \mathcal{B}^{DK}_{00}, \mathcal{B}^{DK}_{0-})$ and $(\mathcal{B}^{DK^*}_{+-}, \mathcal{B}^{DK^*}_{00}, \mathcal{B}^{DK^*}_{0-})$ given in (1), (2) and (3). The relevant CKM matrix elements are taken as $|V_{cb}| = 0.041$ and $|V_{us}| = 0.222$ [9]. We arrive at

$$
\begin{aligned}
T_{DK} &\approx 1.9 \times 10^{-5}\,\mathrm{GeV} \; , \\
T'_{DK} &\approx 3.7 \times 10^{-6}\,\mathrm{GeV} \; ,
\end{aligned}
\tag{15}
$$

and[2]

$$
\begin{aligned}
T_{DK^*} &\approx 2.6 \times 10^{-5}\,\mathrm{GeV} \; , \\
T'_{DK^*} &\approx 4.5 \times 10^{-6}\,\mathrm{GeV} \; .
\end{aligned}
\tag{16}
$$

One can see that $T'_{DK}/T_{DK} \sim T'_{DK^*}/T_{DK^*} \sim 0.2$ holds. Such an instructive and model-independent result is essentially consistent with the naive expectation for T'_{DK}/T_{DK} and T'_{DK^*}/T_{DK^*} ($\sim a^{\rm eff}_2/a^{\rm eff}_1 \approx 0.25$ [2]) in the factorization approximation.

4 SU(3) symmetry breaking

We have pointed out that the $B \to DK$ and the $B \to D\pi$ transitions belong to the same category of exclusive B decays with $|\Delta B| = |\Delta C| = 1$. They can be related to each other under SU(3) flavor symmetry [14]. To examine how good (or bad) this SU(3) symmetry is, let us write out the amplitudes of the $\bar{B}^0_d \to D^+\pi^-$, $\bar{B}^0_d \to D^0\pi^0$ and $B^-_u \to D^0\pi^-$ decays in terms of two isospin amplitudes $A^{D\pi}_{3/2}$ and $A^{D\pi}_{1/2}$:

$$
\begin{aligned}
A^{D\pi}_{+-} &= \frac{1}{\sqrt{3}} A^{D\pi}_{3/2} + \frac{\sqrt{2}}{\sqrt{3}} A^{D\pi}_{1/2} \; , \\
A^{D\pi}_{00} &= \frac{\sqrt{2}}{\sqrt{3}} A^{D\pi}_{3/2} - \frac{1}{\sqrt{3}} A^{D\pi}_{1/2} \; , \\
A^{D\pi}_{0-} &= \sqrt{3} A^{D\pi}_{3/2} \; .
\end{aligned}
\tag{17}
$$

[1] Detailed reanalyses of the $B \to DK$ and $B \to DK^*$ transitions in the factorization approximation and in various form-factor models will be presented in a more comprehensive article [13]. However, some concise discussions can be found in Sect. 4 to phenomenologically understand the effects of SU(3) symmetry breaking in the factorization approximation

[2] Note that the polarization effect of K^* in the final states of $B \to DK^*$ decays has been included in T_{DK^*} and T'_{DK^*}. This point will become transparent in (24), where T_{DK^*} and T'_{DK^*} are calculated in the factorization approximation

Note that these three decay modes can in general occur via three topologically distinct tree-level quark diagrams [11]: the spectator diagram ($\propto V_{cb}V_{ud}^* T_{D\pi}$) similar to Fig. 3a, the color-suppressed diagram ($\propto V_{cb}V_{ud}^* T'_{D\pi}$) similar to Fig. 3b, and the W-exchange diagram ($\propto V_{cb}V_{ud}^* T''_{D\pi}$). In comparison with $T_{D\pi}$ and $T'_{D\pi}$, $T''_{D\pi}$ is expected to have strong form-factor suppression [15]. It is therefore safe, at least to leading order, to neglect the contribution of $T''_{D\pi}$ to the overall amplitudes of $B \to D\pi$ decays. In this approximation, we have

$$A_{3/2}^{D\pi} = \frac{1}{\sqrt{3}} V_{cb}V_{ud}^* \left(T_{D\pi} + T'_{D\pi}\right) e^{i\phi_{3/2}} ,$$
$$A_{1/2}^{D\pi} = \frac{1}{\sqrt{6}} V_{cb}V_{ud}^* \left(2T_{D\pi} - T'_{D\pi}\right) e^{i\phi_{1/2}} , \quad (18)$$

where $\phi_{3/2}$ and $\phi_{1/2}$ are the strong phases of $I = 3/2$ and $I = 1/2$ isospin channels. If SU(3) were a perfect symmetry, one would get $T_{D\pi} = T_{DK}$ and $T'_{D\pi} = T'_{DK}$.

Similar to (14), the explicit expressions of $T_{D\pi}$ and $T'_{D\pi}$ can be obtained in terms of the branching fractions of $\bar{B}_d^0 \to D^+\pi^-$ ($\mathcal{B}_{+-}^{D\pi}$), $\bar{B}_d^0 \to D^0\pi^0$ ($\mathcal{B}_{00}^{D\pi}$) and $B_u^- \to D^0\pi^-$ ($\mathcal{B}_{0-}^{D\pi}$). Then we arrive at the ratios $T_{DK}/T_{D\pi}$ and $T'_{DK}/T'_{D\pi}$ as follows:

$$\frac{T_{DK}}{T_{D\pi}} = \frac{3}{2}\left|\frac{V_{ud}}{V_{us}}\right| \sqrt{\frac{p_{D\pi}}{p_{DK}}} \times \frac{\sqrt{\kappa \mathcal{B}_{0-}^{DK}} + \sqrt{2\left(\mathcal{B}_{+-}^{DK} + \mathcal{B}_{00}^{DK}\right) - \kappa \mathcal{B}_{0-}^{DK}}}{\sqrt{\kappa \mathcal{B}_{0-}^{D\pi}} + \sqrt{6\left(\mathcal{B}_{+-}^{D\pi} + \mathcal{B}_{00}^{D\pi}\right) - 2\kappa \mathcal{B}_{0-}^{D\pi}}} ,$$
$$\frac{T'_{DK}}{T'_{D\pi}} = \frac{3}{2}\left|\frac{V_{ud}}{V_{us}}\right| \sqrt{\frac{p_{D\pi}}{p_{DK}}} \times \frac{\sqrt{\kappa \mathcal{B}_{0-}^{DK}} - \sqrt{2\left(\mathcal{B}_{+-}^{DK} + \mathcal{B}_{00}^{DK}\right) - \kappa \mathcal{B}_{0-}^{DK}}}{2\sqrt{\kappa \mathcal{B}_{0-}^{D\pi}} - \sqrt{6\left(\mathcal{B}_{+-}^{D\pi} + \mathcal{B}_{00}^{D\pi}\right) - 2\kappa \mathcal{B}_{0-}^{D\pi}}} , \quad (19)$$

where

$$p_{D\pi} = \frac{1}{2M_B} \times \sqrt{\left[M_B^2 - (M_D + M_\pi)^2\right]\left[M_B^2 - (M_D - M_\pi)^2\right]} \quad (20)$$

is the c.m. momentum of the D and π mesons. Typically taking the central values of $\mathcal{B}_{+-}^{D\pi}$, $\mathcal{B}_{00}^{D\pi}$ and $\mathcal{B}_{0-}^{D\pi}$ reported in [9], we obtain

$$\frac{T_{DK}}{T_{D\pi}} \approx 1.21 ,$$
$$\frac{T'_{DK}}{T'_{D\pi}} \approx 0.97 . \quad (21)$$

This model-independent result can be confronted with the result achieved from the naive factorization approximation:

$$\frac{T_{DK}}{T_{D\pi}} = \frac{f_K}{f_\pi} \cdot \frac{F_0^{BD}(M_K^2)}{F_0^{BD}(M_\pi^2)} \approx 1.22 ,$$
$$\frac{T'_{DK}}{T'_{D\pi}} = \frac{M_B^2 - M_K^2}{M_B^2 - M_\pi^2} \cdot \frac{F_0^{BK}(M_D^2)}{F_0^{B\pi}(M_D^2)} \approx 1.09 , \quad (22)$$

where $f_\pi = 130.7\,\mathrm{MeV}$, $f_K = 159.8\,\mathrm{MeV}$ [9], $F_0^{B\pi}(0) = 0.28$ and $F_0^{BK}(0) = 0.31$ [2] have been used. We observe that (21) and (22) are consistent with each other. It is suggestive that the factorization hypothesis may work well for the individual isospin amplitudes of $B \to DK$ and $B \to D\pi$ transitions.

One can analogously compare between the quark-diagram amplitudes of $B \to DK^*$ and $B \to D\rho$ decays, i.e., T_{DK^*} (or T'_{DK^*}) and $T_{D\rho}$ (or $T'_{D\rho}$), to estimate the size of SU(3) flavor symmetry breaking. Typically taking the central value of $\mathcal{B}_{00}^{D\rho}$ reported by the Belle Collaboration [16] and those of $\mathcal{B}_{+-}^{D\rho}$ and $\mathcal{B}_{0-}^{D\rho}$ reported in [9], we obtain

$$\frac{T_{DK^*}}{T_{D\rho}} \approx 1.00 ,$$
$$\frac{T'_{DK^*}}{T'_{D\rho}} \approx 0.70 . \quad (23)$$

In comparison, the factorization approximation yields

$$\frac{T_{DK^*}}{T_{D\rho}} = \frac{M_{K^*}}{M_\rho} \cdot \frac{f_{K^*}}{f_\rho} \cdot \frac{F_1^{BD}(M_{K^*}^2)}{F_1^{BD}(M_\rho^2)} \left|\frac{\epsilon_{K^*} \cdot p_B}{\epsilon_\rho \cdot p_B}\right| = \frac{p_{DK^*}}{p_{D\rho}} \cdot \frac{f_{K^*}}{f_\rho} \cdot \frac{F_1^{BD}(M_{K^*}^2)}{F_1^{BD}(M_\rho^2)} \approx 1.02 ,$$
$$\frac{T'_{DK^*}}{T'_{D\rho}} = \frac{M_{K^*}}{M_\rho} \cdot \frac{A_0^{BK^*}(M_D^2)}{A_0^{B\rho}(M_D^2)} \left|\frac{\epsilon_{K^*} \cdot p_D}{\epsilon_\rho \cdot p_D}\right| = \frac{p_{DK^*}}{p_{D\rho}} \cdot \frac{M_B - 2\sqrt{M_{K^*}^2 + p_{DK^*}^2}}{M_B - 2\sqrt{M_\rho^2 + p_{D\rho}^2}} \cdot \frac{A_0^{BK^*}(M_D^2)}{A_0^{B\rho}(M_D^2)} \approx 1.16 , \quad (24)$$

where $f_{K^*} = 214\,\mathrm{MeV}$, $f_\rho = 210\,\mathrm{MeV}$, $A_0^{BK^*}(0) = 0.47$ and $A_0^{B\rho}(0) = 0.37$ [2] have typically been used. We see that the results for $T_{DK^*}/T_{D\rho}$ in (23) and (24) are in good agreement, but there is a remarkable discrepancy between the model-dependent and model-independent results for $T'_{DK^*}/T'_{D\rho}$.

5 Further discussions

The present experimental data on $B \to DK$ and $B \to DK^*$ decays in (1), (2) and (3) allow us to determine their isospin and quark-diagram amplitudes in a quantitatively meaningful way. It should be noted, however, that the accuracy of $\mathcal{B}_{+-}^{DK^*}$ and $\mathcal{B}_{0-}^{DK^*}$ is rather poor. Hence the result $T'_{DK^*}/T'_{D\rho} \approx 0.7$ in (23) is most likely a signal of poor accuracy of the current data, instead of a signal of significant SU(3) symmetry breaking. As one can see from (19), $T'_{DK^*}/T'_{D\rho}$ may be more sensitive to the uncertainties of $\mathcal{B}^{DK^*}_{+-}$ and $\mathcal{B}^{DK^*}_{0-}$ than $T_{DK^*}/T_{D\rho}$, because the former involves large cancellations in both its numerator and denominator. We therefore argue that the existing discrepancy between the model-dependent and model-independent results of $T'_{DK^*}/T'_{D\rho}$ could essentially disappear, when more precise measurements of $B \to DK^*$ decays are available.

It is worth remarking that only the central values of the relevant branching fractions of the $B \to DK^{(*)}$ decays have been taken into account in our numerical estimates of the quark-diagram amplitudes and SU(3) symmetry breaking effects. In addition, current experimental error bars associated with a few of those branching fractions remain too large to allow for a firm quantitative conclusion about the size and phase of final-state rescattering in every $|\Delta B| = |\Delta C| = |\Delta S| = 1$ mode. The same point has been emphasized by Chiang and Rosner in [12]. Hopefully, more accurate data will soon be available at B-meson factories.

In general, the $B \to DK^{(*)}$ decays include not only $\bar{B}^0_d \to D^+ K^{(*)-}$, $\bar{B}^0_d \to D^0 \bar{K}^{(*)0}$ and $B^-_u \to D^0 K^{(*)-}$ but also $\bar{B}^0_d \to \bar{D}^0 \bar{K}^{(*)0}$, $B^-_u \to \bar{D}^0 K^{(*)-}$ and $B^-_u \to D^- \bar{K}^{(*)0}$. So far only the former have been measured at B-meson factories. The latter have lower branching fractions, because of the stronger CKM suppression: $|V_{ub}V^*_{cs}|^2/|V_{cb}V^*_{us}|^2 \approx 0.15$ [9]. Note that the isospin relations among three amplitudes of $\bar{B}^0_d \to \bar{D}^0 \bar{K}^{(*)0}$, $B^-_u \to \bar{D}^0 K^{(*)-}$ and $B^-_u \to D^- \bar{K}^{(*)0}$ transitions are quite similar to those given in (4) [10]. In particular, the unknown branching fractions of $\bar{B}^0_d \to \bar{D}^0 \bar{K}^0$ ($\tilde{\mathcal{B}}^{DK}_{00}$) and $\bar{B}^0_d \to \bar{D}^0 \bar{K}^{*0}$ ($\tilde{\mathcal{B}}^{DK^*}_{00}$) can be predicted from the measured branching fractions of $B^-_u \to D^0 K^-$ and $B^-_u \to D^0 K^{*-}$. The results are

$$\begin{aligned} \tilde{\mathcal{B}}^{DK}_{00} &= \kappa \left| \frac{V_{ub}V^*_{cs}}{V_{cb}V^*_{us}} \right|^2 \left(\frac{T'_{DK}}{T_{DK} + T'_{DK}} \right)^2 \mathcal{B}^{DK}_{0-} \\ &\approx 1.4 \times 10^{-6} \, , \\ \tilde{\mathcal{B}}^{DK^*}_{00} &= \kappa \left| \frac{V_{ub}V^*_{cs}}{V_{cb}V^*_{us}} \right|^2 \left(\frac{T'_{DK^*}}{T_{DK^*} + T'_{DK^*}} \right)^2 \mathcal{B}^{DK^*}_{0-} \\ &\approx 1.8 \times 10^{-6} \, , \end{aligned} \tag{25}$$

well below the experimental upper bounds reported by the Belle Collaboration [8]. If the phase shift between the $I = 1$ and $I = 0$ isospin channels in $B^-_u \to \bar{D}^0 K^{(*)-}$ and $B^-_u \to D^- \bar{K}^{(*)0}$ transitions is assumed to equal the corresponding phase shift between the $I = 1$ and $I = 0$ isospin channels in the $\bar{B}^0_d \to D^+ K^{(*)-}$ and $\bar{B}^0_d \to D^0 \bar{K}^{(*)0}$ decays[3] [10,17], then instructive predictions can be obtained for the branching fractions of $B^-_u \to \bar{D}^0 K^{(*)-}$ ($\tilde{\mathcal{B}}^{DK}_{0-}$ or $\tilde{\mathcal{B}}^{DK^*}_{0-}$) and $B^-_u \to D^- \bar{K}^{(*)0}$ ($\tilde{B}^{DK}_{-0}$ or $\tilde{B}^{DK^*}_{-0}$):

$$\begin{aligned} \tilde{\mathcal{B}}^{DK}_{0-} &= \frac{1}{\kappa} \tilde{\mathcal{B}}^{DK}_{00} \cos^2 \frac{(\phi_1 - \phi_0)_{DK}}{2} \\ &\approx 1.2 \times 10^{-6} \, , \\ \tilde{\mathcal{B}}^{DK^*}_{0-} &= \frac{1}{\kappa} \tilde{\mathcal{B}}^{DK^*}_{00} \cos^2 \frac{(\phi_1 - \phi_0)_{DK^*}}{2} \\ &\approx 1.8 \times 10^{-6} \, , \end{aligned} \tag{26}$$

and

$$\begin{aligned} \tilde{\mathcal{B}}^{DK}_{-0} &= \frac{1}{\kappa} \tilde{\mathcal{B}}^{DK}_{00} \sin^2 \frac{(\phi_1 - \phi_0)_{DK}}{2} \\ &\approx 2.7 \times 10^{-7} \, , \\ \tilde{\mathcal{B}}^{DK^*}_{-0} &= \frac{1}{\kappa} \tilde{\mathcal{B}}^{DK^*}_{00} \sin^2 \frac{(\phi_1 - \phi_0)_{DK^*}}{2} \\ &\approx 1.8 \times 10^{-7} \, . \end{aligned} \tag{27}$$

In arriving at these model-independent results, we have neglected small contributions of the annihilation-type quark diagrams to the $B^-_u \to \bar{D}^0 K^{(*)-}$ and $B^-_u \to D^- \bar{K}^{(*)0}$ decays. Although both charged modes have quite small branching fractions, they are worth being searched for at B-meson factories. The reason is simply that the transitions under discussion are very important for a relatively clean determination of the weak angle γ of the well-known CKM unitarity triangle [5].

In summary, we have for the first time determined the isospin amplitudes and their relative phase for the $B \to DK^{(*)}$ transitions by the use of current experimental data. It is found that the isospin phase shifts in $B \to DK$ and $B \to DK^*$ decays are very likely to lie in the ranges $37^\circ \leq (\phi_1 - \phi_0)_{DK} \leq 63^\circ$ (or around 50°) and $25^\circ \leq (\phi_1 - \phi_0)_{DK^*} \leq 50^\circ$ (or around 35°), although the possibility $(\phi_1 - \phi_0)_{DK} = (\phi_1 - \phi_0)_{DK^*} = 0^\circ$ cannot be excluded at present. Thus significant final-state rescattering effects possibly exist in such exclusive $|\Delta B| = |\Delta C| = |\Delta S| = 1$ processes. We have also analyzed the $B \to DK^{(*)}$ transitions in the language of quark diagrams, and determined the amplitudes of their spectator and color-suppressed spectator diagrams. We stress that our results are model-independent. The SU(3) flavor symmetry is examined by comparing the quark-diagram amplitudes of the $B \to DK$ and $B \to DK^*$ decays with those of the $B \to D\pi$ and $B \to D\rho$ decays. We find that the effects of SU(3) symmetry breaking are in most cases understandable in the factorization approximation, which works for the individual isospin amplitudes. Finally, very instructive predictions for the branching fractions of $\bar{B}^0_d \to \bar{D}^0 \bar{K}^{(*)0}$, $B^-_u \to \bar{D}^0 K^{(*)-}$ and $B^-_u \to D^- \bar{K}^{(*)0}$ transitions have been presented. We expect that more precise experimental data from B-meson factories will help us to gain a deeper understanding of the dynamics of $B \to DK^{(*)}$ decays and to probe the signal of CP violation in them.

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[3] Such an assumption has been questioned in [18], but there have not been strong theoretical arguments to disprove it. The measurements of $B^-_u \to \bar{D}^0 K^{(*)-}$ and $B^-_u \to D^- \bar{K}^{(*)0}$ decays in the future at B-meson factories will allow us to clarify the present ambiguity associated with their isospin phases

References

1. BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. **89**, 201802 (2002); Belle Collaboration, K. Abe et al., Phys. Rev. D **66**, 071102 (2002)
2. For recent reviews with extensive references, see M. Neubert, B. Stech, Adv. Ser. Direct. High Energy Phys. **15**, 294 (1998); M. Beneke et al., Nucl. Phys. B **591**, 313 (2000)
3. R. Fleischer, Phys. Rept. **370**, 537 (2002), and references therein
4. Z.Z. Xing, Phys. Lett. B **364**, 55 (1995); I. Dunietz, Phys. Lett. B **427**, 179 (1998)
5. M. Gronau, D. London, Phys. Lett. B **253**, 483 (1991); M. Gronau, D. Wyler, Phys. Lett. B **265**, 172 (1991); D. Atwood, A. Soni, hep-ph/0212071
6. Belle Collaboration, K. Abe et al., Phys. Rev. Lett. **88**, 052002 (2002); CLEO Collaboration, T.E. Coan et al., Phys. Rev. Lett. **88**, 062001 (2002)
7. See, e.g., Z.Z. Xing, hep-ph/0107257; M. Neubert, A.A. Petrov, Phys. Lett. B **519**, 56 (2001); H.Y. Cheng, Phys. Rev. D **65**, 094012 (2002)
8. Belle Collaboration, P. Krokovny et al., hep-ex/0212066
9. Particle Data Group, K. Hagiwara et al., Phys. Rev. D **66**, 010001 (2002)
10. N.G. Deshpande, C.O. Dib, Phys. Lett. B **319**, 313 (1993); Y. Koide, Phys. Rev. D **40**, 1685 (1989)
11. Z.Z. Xing, J.Z. Lu, Nuovo Cim. A **109**, 549 (1996)
12. I thank J.L. Rosner for emphasizing this point to me (also see C.W. Chiang, J.L. Rosner, hep-ph/0212274, version 3, January 2003)
13. Q. Wang, Z.Z. Xing, in preparation
14. See, e.g., M. Gronau, O.F. Hernández, D. London, J.L. Rosner, Phys. Rev. D **52**, 6374 (1995)
15. See, e.g., Z.Z. Xing, Phys. Rev. D **53**, 2847 (1996); B. Blok, M. Gronau, J.L. Rosner, Phys. Rev. Lett. **78**, 3999 (1997)
16. Belle Collaboration, A. Satpathy et al., hep-ex/0211022
17. Z.Z. Xing, Phys. Rev. D **58**, 093005 (1998)
18. M. Gronau, J.L. Rosner, Phys. Lett. B **439**, 171 (1998)